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ABSTRACT

This paper begins by discussing the observation that mathematics instruction at all levels is in a crisis of relevance. A sketch of the function and organization of the existing mathematics instruction is presented; the suggestion that instruction be built on a problem-orientation basis is considered; and the role and education of the teacher is discussed. The paper concludes with a description of the new mathematics teacher education program (at the elementary school level) having problem-oriented project work as its focus, which has been established at Roskilde University Centre in Denmark. (DT)

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THE "CRISIS" IN MATHEMATICS INSTRUCTION AND A NEW TEACHER EDUCATION AT GRAMMAR SCHOOL LEVEL

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Abstract

The present article takes its starting point in the observation, affirmed by several people, that mathematics instruction at all levels is in a "crisis", a crisis of relevance.

The paper argues - after having presented a sketch of the function and organization of the existing mathematics instruction - that a necessary condition for the crisis to be dissolved is that the role of mathematics undergoes a radical change, such as that instruction be built up on the basis not of a puzzle-like but a proper problem-orientation, where real, complex non-mathematical problems are subject to mathematical treatment.

These considerations are followed up by a valuation of the role and the education of the teacher, concluding that the teacher himself in his education must have worked with such real, complex problems and not only the theories and methods of mathematics in itself.

The paper is concluded with a description of a new mathematics teacher education programme (at grammar school level), which has proper, problem-oriented project-work in its focus. This education programme has been established at Roskilde University Centre in Denmark, following recommendations from a committee (set down by the Ministery of Education) of which the author of this article was the chairman.

THE "CRISIS" IN MATHEMATICS INSTRUCTION AND A NEW TEACHER EDUCATION AT GRAMMAR SCHOOL LEVEL

A: BACKGROUND

I. Introduction: "Crisis" in mathematics instruction

Perhaps it is not sufficiently justified to talk about the existence of a crisis in mathematics instruction in the primary school, in the secondary school and at university level. If, however, it is not sufficiently justified to talk about a crisis - a concept the use of which presupposes that the state of affairs has formerly been better - it is undeniable that everything is not as it ought to be. Reports from different quarters (see for instance Burghes [5], Cornelius [6], Selkirk [13])x agree in the observation that a considerable part of the students in schools, universities and other institutions of higher education have difficulties in understanding the character and content of mathematics, to find its results relevant in a broader context than that of mathematics itself. The students fear the subject (even if they have chosen it as an optional at school or as a major at university (Selkirk, Cornelius)). They are bored, work either too much, without really getting anywhere, or too little but strained by a constant bad conscience and a permanent feeling of insecurity.

Whether these symptoms have existed before but have remained unnoticed, or they have shown up only in the last few years - in which case it is well justified to talk about a crisis - it is necessary, in order to find a solution, to pay attention to three questions:

1. What are the actual components of the observed "crisis"?

x) Also compare the following quotation from Imre Lakatos in [9] , which is from 1963: "..science and mathematics teaching is disfigured by the customary authoritarian presentation. Thus presented knowledge appears in the form of infallible systems hinging on conceptual frameworks not subject to discussion. The problemsituational background is never stated and is sometimes already difficult to trace. Scientific education - atomized according to separate techniques - has degenerated into scientific training. No wonder that it dismays critical minds."

2. What is the role of mathematics in society and what could it be?

3. How should - as a consequence of answers to 1. and 2. - mathematics instruction work

Up till now reflections in the literature on the problems have to a large extent concentrated on 1., and here most of all on the relation to school mathematics and more specifically mathematics in the primary school. Som authors have pondered on whether the nature of mathematics is such that only persons with specific and rare gifts can have success in working with it (Thom [16]). Others have pointed out the trouble to lie not in the mathematical substance "an sich" but in the requirements to stringency and precision in exposition and proofs and in a charging (and new) terminology and use of symbols. In addition it has been asked if the organization of the teaching situations could be charged with the main responsibility: is the curriculum too large? - is the speed too fast? - are the teachers boring? - are the examples too casual? - do the students possess suitable prerequisites from their previous education? - have applications been too much neglected?

Common to all these attempts at an explanation - attempts that all touch on a part of the truth - is the fact that neither the role of mathematics (the way in which it becomes actualized in non-mathematical surroundings) nor its body (the fabric and material of its disciplines) have been discussed very deeply. Although one can find discussions about the advantages of geometry compared to algebra (Thom [16]), or of algebra compared to probability theory, or where the fronts might go, these discussions have most often left the role and body of mathematics untouched. Things are presented as if the problem consisted in determining a curriculum-content and in finding out in what way this content should be served in order to make it digestible.

Let it be said ven now that I hold it necessary also to take into account the role and body of mathematics (in the above-

sketched sense) as a factor that must be changed if the crisis of mathematics instruction is to be solved. (A passing remark: In a way the mere usage "to solve the crisis of mathematics instruction" suggests that what the crisis deals with is the instruction of mathematics, not mathematics itself; that mathematics itself is sufficiently sound and relevant but temporarily in trouble with its surroundings. Efforts at solutions based on the conception that the only way in which the role and body of mathematics can be changed is in the choice and presentation of curriculum-content, can only lead to "technical"-teaching - in a broad sense of the word "technical" - and patching, instead of proper solutions.)

Mathematics must be looked upon as a part of a larger whole: related to other subjects, to problems of forecasting, planning and control in society, in everyday practice etc., and not so narrowly as has been the case up till now within the horizon of mathematics on its own (or, at a pinch, of physics).

We shall return to these points later in this article (section V).

II. The organization of the existing mathematics instruction

Let us begin by looking a little closer at mathematics instruction as it appears to be practiced in several places at grammar school level and in the first years at the universities and other institutions of higher education. The description has - in order to be founded on concrete matters - Danish circumstances as its object, but probably most of the features mentioned can also be found (in varying combinations and modifications) in other countries[x].

Students meet with - as the only thing - a well-defined mathematics curriculum-content, upon which they have only minimal

x) A brief, but useful survey over the character of school mathematics and its reforms in a number of continental european countries can be found in Servais [14].

influence. And to the extent that such an influence does exist it is limited to the possibility of choosing between certain topics under certain restrictions.

The teaching in a given topic is organized along an outline that very closely follows the arrangement of the textbooks used:

A short introduction to the specific topic - introductory fixing of new concepts - statement of rigorous definitions - presentation of a few almost trivial examples to illustrate the definitions - formulation and demonstration of a collection of lemmas and propositions - formulation and demonstration of main theorems (if there are any) - a more careful presentation of more substantial examples - possibly (but far from always) discussion of the application of the established concepts and results in a few simpler and usually constructed examples of non-mathematical, mostly physical, origin - solution of exercises in order to drill familiarity with the concepts and train proficiency in internally mathematical problem-solving.

The teaching does not necessarily comprise all the elements of the sequence (especially not the first and the last ones), just as there might appear elements not mentioned here.

Teaching forms alternate between presentation by the teacher of new textbook material (in the universities in separate lectures), examination of the students (mainly in the school) in the preceding material, discussion about difficult points in the text, exercise-solving - extemporal or on the basis of students' preparation - at the blackboard. In universities and other institutions of higher education that part of the teaching not consisting of lectures is taken care of by instructors. Sometimes there can be seen elements of group work, chiefly with exercise-solving.

The topics are all purely mathematical in character and are

built up with the use of the terminology and symbols of modern mathematics and under observation of consistency and stringency. A great emphasis is laid on an "economical" presentation of the material, i.e. concepts are presented in detail only once, after which they in any later stage are presupposed to be known by the students.

Thus mathematics assumes the shape of a coherent hierarchical system. The exposition x) emphasizes a deductive formal-logical progression without paying much attention - apart from in footnotes and passing remarks - to historical or outward turned aspects of the content and development of the subject. As Brolin and Pleijel put it in [3]: "However, from the point of view of science philosophy the deductive-axiomatic character of the subject stands as the most important feature of the mathematical education in school."

One of the striking points in this instruction is that reality - represented by examples - serves to illustrate mathematics, at the most to demonstrate its power to assist in handling certain problems from physics. What is given is mathematics. Its structures, operations and rules constitute the object for study. The examples serve, whether they are constructed, academis examples with only a thin "varnish of reality", or more realistic but simplified ones, to motivate and facilitate the learning. They have only seldom an interest in their own right.

The students having success xx) in such an instruction may be of two sorts: Either those who can find themselves satisfied by the abstract manipulation of formal concepts, and who regard the problems of legitimacy of mathematics towards the outside world to be either uninteresting or solved by the exist-

x) The exposition is of the type, that by Glaeser in [8] is called "la pédagogie d'exposition" and which has been dominant most of this century.

xx) Compare Glaeser in [8]:"l'excellent élève est celui qui a décodé (et qui saura décoder ultérieurement) une grande masse de connaissance présentées chaque fois dans la version correcte."

ence of a few examples or by invocation of an obvious relevance not subject to discussion. Or the - negligibly few - students being capable not only of acquiring the formal, deductive apparatus but also of looking through its construction and of understanding some of the origin, development and role of concepts and theories. The vast majority of the students must resign themselves to understanding the edifice of mathematics at a micro-level - and actually only a minority reach that far - that is to understand the sequences of arguments step by step, but not in entireties and main points, and perhaps also to acquire a proficiency in solving mathematical exercises that are neither too sophisticated nor connected with reality in too complicated a way.

A (perhaps a little programmatically formulated) characteristic of this instruction: It is possible that mathematics instruction is not being given for its own sake, but it appears as if it were. Reality serves to illustrate mathematics, not conversely. The relevance of mathematics is neither subject to nor expressed in the instruction but is supposed to be established beforehand and outside the instruction itself.

This is - I believe - one of the most essential components of the crisis discussed, which perhaps is best called a crisis of relevance[x]. It is a necessary condition for a dissolution of the crisis that mathematics instruction - and research in mathematics too, for that matter! - is brought to relate itself to reality and its problems in a far more direct and not merely legitimacy-seeking way than is the case. Mathematics must leave its introvert inbreeding and actively demonstrate its relevance to essential problems from the non-mathematical sphere. Let us quote Thom [16]:"No one can reasonably escape the impression that the most important mathematical structures (algebraic structures, topological structures) appear as fundamental data imposed by the exterior world, and that their irrational diversity finds its only justification in reality."

x) It is characteristic that the word "relevance" appears as the "key word" (Sida [15]) in many of the contributions to the debate on innovation in mathematics education. See for instance also Ford and Hall [7] and Ormell [12] .

In sections VI and VII is discussed what consequences for the mathematics teacher education the existing mathematics instruction has and what a new one ought to have.

III. The function of the existing mathematics education

In the school system mathematics education at grammar school level has traditionally served two objectives, which have not always been very explicit although probably deducible from the actual content and character of the instruction:

a. to prepare for further studies, for a mathematics study proper as well as for studies where mathematics is a supporting subject

b. to be a means for intellectual - primarily logical, secondarily aesthetic and personal - schooling.

These objectives can to a certain extent be served by a mathematics instruction resting on a selection of purely mathematical topics with a building-up and exposition on the premisses of modern mathematics. This non-antagonistic relationship between the objectives and organization of mathematics education certainly does not suffice to remove students' difficulties in acquiring the subject - even if they accept the objectives. It gives, however, a little of the explanation of why reflections on the possible backgrounds of the crisis discussed have been concerned almost solely with point 1 in section I, and why the search for a solution has chiefly left the role and body of mathematics untouched.

The (co-operating) reasons why the crisis-like features manifest themselves so clearly and so often these years and why they - as diagnosed above - assume the shape of a crisis of relevance, are particularly the following two:
(1) Society has changed from a society in which relatively few go through and thence prepare themselves for a higher education leading - at least in Denmark - to a relatively small amount of well-defined professions, to a society marked by the

qualitative and quantitative expansion which from the beginning of the sixties set in over the university sphere over most of the western world and which so far has put a decisice stamp on all higher education and the preparation for it at secondary school. This implies that mathematics instruction is now being given to a band of students with a far broader spectrum of interests and potential future professions than before.

(2) In the course of the sixties strong demands were released from student youth that studies should play a socially relevant/ critical role - a role that could not be taken care of by the subjects in their contents and forms as known so far.

Initially the expansion chiefly concerned the technological and natural science fields where particular emphasis was laid on bringing instruction and hence also mathematics instruction more into line with the progress - not to confuse with the process- of science than it had been in the preceding period x).

Mathematics education, however, has changed its function. The bringing of instruction into line with modern mathematics as a science is no longer sufficient to supply the demand from society for a more differentiatedly educated labour force with a certain competence in mathematics but not necessarily with interests in pure mathematics. Earlier, the application of more advanced mathematical theories and methods was in the hands of physicists and engineers. As things have developed, however, mathematics - as several authors have called attention to, for instance Bleyer et alt. [1] , Ford and Hall [7] , Lee [10] and Sida [15] - is playing a still wider role, also in its more advanced forms. This is the case not only with various sciences (for instance linguistics, data processing, biology, economics, geography etc.) but also with a series of other professions in society (technicians, planners, administrative personel in public administration and business etc.).Even the citizen who from the point of view of mathematics is a layman, and who perhaps in his daily work does not have much contact with mathematics,

x) Compare May [11]: "The reform of the mathematical curriculum has so far concentrated on modernization of content by introducing more modern ideas and by giving theoretical considerations heavier weight."

needs to be able to make up his mind about a lot of real problems with a woof of mathematics: assessment of the consequences for income distribution of specific changes in tax legislation, political-economical long range plans, forecasts of population growth, development in emplyment, consumption of resources (energy resources, for instance), besides a lot of statistical problems.

These demands to the qualifications of the labour force and citizens must, of course, come through in the instruction at school - a tendency that,even if as yet to a small extent, is going to be seen, for example, at grammer school level in Denmark. These changes imply that mathematics instruction - and not only instruction as a whole - must take other matters, besides those of mathematics itself, under consideration.

IV The function of a changed mathematics education

Without going into a detailed disussion of the aims and characteristics of mathematics teaching under the described altered conditions, I shal state a few main points which are components of the function of mathematics at grammar school:

The generally qualifying function of mathematics education

Mathematics must be a means for students (and later adults) in structuring real, complex, non-mathematical problems in categories accessible to mathematical treatment. The structuring serves to translate problems from reality into such questions in mathematics (mathematical models) as can be treated by students (and later adults) either directly on the basis of their existing insight and skill or after supplementing the necessary prerequisites.

This function is equally important whether it concerns a study- or profession-oriented occupation with mathematics or "only" a social qualification of the non-mathematician. This is not least important as regards the produvtive power of mathematical insight, i.e. the power of producing new tools, facing new problems.

In the generally qualifying function of mathematics I also include the preparative mathematical qualification of students who are later to study other subjects than mathematics.

The cultural function of mathematics education

Mathematics instruction must, in a critical way, effect understanding of those aspects of theory of knowledge which have to do with the role (and limitations) of logical and deductive systems in carrying out scientific inference and formulating scientific theories. Here mathematics is to serve not only as an abstract textbook example but also to promote a deepened understanding of the deductive features of reality based on mathematical models. Thus the proper function of mathematics is to be a transformer between reality and models of it.

Mathematics education as a background for the development of mathematics as a science

This function is the most traditional of the three. On the other hand it is perhaps less traditional to claim that the development of mathematics as a science - in the light of social needs for mathematical knowledge - is not necessarily promoted acceptably by continued activity inside the strongly specialized mathematical disciplines. The opening towards non-mathematical problems implies new demands for scientific occupation with developing mathematics[x)].

These few remarks serve to stress that the generally qualifying function of mathematics is also to a large extent relevant to any person who is to work with the progression of science and /or with teaching. Let me support thesis 8 (p.28) in Bleyer et alt. [1] :"The distinction between "pure" and "applied" mathematics corresponds to a socially determined internal mathematical division of labour. The university

x) This holds even if one with Ormell [12]claims that "there is no fundamental dividing line between so-called "pure" and so-called "applied mathematics"". One cannot apply (nor does Ormell) this - as far as I can see: valid - science-philosophical finding to legitimate all pure mathematics independentlyof what it is concerned with.

must make the professional mathematician as well as the teacher familiar with the totality of the working-process in mathematics with its four main components if a false picture of the social tasks of mathematics is not to be induced: The student of mathematics must, if possible in the fundamental education, learn - taking a given problem in society as a starting point - to judge its mathematical solubility, to make it precise, to model, to systematize it and to find its numerical solution. The suitable form for this is project-work."[x)]

V. The organization of a changed mathematics education

An instruction that aims at fulfilling the three above-mentioned functions should make students capable of building mathematical models for non-mathematical phenomenae [xx)] and of carrying through a solving procedure - resting on a coherent foundation - for problems translated from reality to a model. The teaching must therefore have its starting point in real, complex, non-mathematical problems. The word"real" is to be taken literally, the problems are not to be thought of as puzzle-problems.

The following aspects should be taken into consideration:

Ideally, the problems from reality must determine which parts of mathematics are to come into play, and not conversely. Nevertheless the relationship between reality on the one hand and mathematics on the other must be regarded as dialectical and not

x) Translated from German: "Die Trennung von"reiner" und "angewandter" Mathematik entspricht einer gesellschaftlich überholten innermathematischer Arbeitsteilung, Die Universität muss den Diplommathematiker wie den Lehrer auf die Totalität des Mathematische Arbeitsprozesses mit seinen vier Hauptmomenten vorbereiten, wenn sie ihm nicht ein falsches Bild von der gesellschaftlichen Aufgaben der Mathematik vermittelt will. Der Mathematikstudent muss möglichst schon in der Grundausbildung lernen, ausgehend von einem gegebenen gesellschaftichen Problem, dessen mathematische Lösbarkeit abzuschätzen, die Aufgabestellung zu präzisieren, zu modellieren, zu systematisieren und das numerische Lösungsverfahren zu bestimmen. Die geignete Form dafür ist das Projektstudium."

xx) This perspective is, as it will be known, not original. Apart from Bleyer at alt., quoted above, Ford and Hall [7] , Sida [15] , Lee [1o], Brown [4] , and Braunfeld and Kaufmann [2] can be referred to. On the other hand it is important to underline that what is said in the present article refers to all mathematics and not only(as in Fordand Hall) to apllied mathematics. So I am in agreement with Ormell [12] at this point.

static/ideal.

First of all the real problems subject to treatment must be pointed out by means of some criteria. Since not any problem can directly be put through an appropriate mathematical treatment, and since it often - in the cases where such a treatment is possible - is necessary, furthermore, to apply a mathematical apparatus requiring many prerequisites, it is obvious that the selection of problem-situations for teaching cannot take place exclusively according to an assessment of the relevance and interest of the problems. The selection must rest also on the actual mathematical insight of the students and on their possibilities of acquiring, in a practicable way, a knowledge about mathematical topics relevant to the handling of the problem and with which they were not familiar from the outset.

Secondly the chosen problem will often in its first form be so diffuse and contain so mane aspects and subproblems that it cannot undergo a more than superficial treatment in this form. It must, therefore, be narrowed down. The process of narrowing down is, of course, partly determined by which features of the original problem are considered the most important in the situation, and which - again - partly - by the mathematical apparatus which is present, or might be built up, in the students' minds.

This dialectic process, in which, on the one hand, the students' mathematical apparatus is built up by the treatment of real, non-mathematical problems, and which, on the other hand, the selection and narrowing down of such real problems depend on the manifest or latent possibilities of the students, is a voyage between two extremes, each of which lacks one of the fundamental aspects of dialectics, which, therefore, is non-existing at these extremes.

The first extreme appears, if,initially, for one reason or another, a fixed definite curriculum-content is given, which the

students have to learn. In this case the possibilities of choosing and narrowing down problems are limited to dealing with the problems from which the students can learn the fixed content. If the curriculum-content is extensive, in relation to the total amount of time at disposal, that aspect of the dialectics, in which the choice of mathematical topics is determined by non-mathematical problems is, in fact, dissolved. In this way the role of such problems in instruction is reduced to being almost exclusively of a motivating character, although this motivation be both genuine and effective. If the restrictions on curriculum-content are so tight that almost all the time at disposal is needed for mere learning of the material, the role of problems degenerates to providing examples and illustrations.

The second extreme appears if such a strong priority is acribed to the problems at issue, for instance by requirements from other subjects, that not much attention is paid to whether or not the mathematical prerequisites and the possibilities for the students of acquiring knowledge about the topics needed for the actual purpose are, in fact, present. So the possibilities for the students of handling the problems are at best reduced to the applying of undigested second-hand methods and results on the directions of the teacher - and the dialectics has disappeared.

It is obvious that if the process in question is to run successfully, the teacher must possess such a breadth of outlook that he can judge what mathematical apparatus could be brought up in the treatment of a given problem, as well as whether the necessary mathematics is present or can be built up with the students.

These reflections have taken these proportions, because "problem-orientation" has been a key word in the debate about the difficulties in mathematics teaching and because it seems that the word is used in many different meanings. These meanings, however, have almost all referred to <u>internal</u> mathematical problems or at pinch game problems, problems from mechanics

and the like, where the construction of models is often fairly simple. This holds even if emphasis is laid on ambiguously formulated, open problems, or on the generation of problems by the students themselves (Wittmann [17]). In my opinion an internal concept of problem-orientation is not sufficient to start the dialectical process discussed.

The reason why it is reasonable to submit certain problems to a mathematical treatment at all is, of course, that the handling of the problems cannot - or only with much greater difficulties - take place without mathematical aid. The mathematics instruction must also, therefore, as an object have mathematical structures and problems formulated within such structures. The important thing is that these structures come into play as a result of the needs originated in the working up of the problems at issue and in respecting the dialectics pointed to above. This principle of structuring stands in opposition to the traditional one (which can be briefly summarized like this: experience shows that is worth while learning such and such skills, since they have proved useful for a lot of purposes, even if at the moment of learning they seem to be arbitrary and without external motivation) is undoubtedly corroborated in psychological research. The effect of learning is much bigger if it has its starting point in problems relevant to the learner - especially if they are formulated by him/her.

The mathematical treatment of a non-mathematical problem must aim both at solving the problem itself and at producing insight that can be mobilized in other problem-situations. So the insight must be raised over the concrete situation with its specific restrictions, to an understanding of the structure of the problem. This understanding, however, must be anchored in concrete matters and its mathematical treatment must find inspiration from them.

Perhaps there, finally, is a good reason for stressing that the described view on, and practice of, mathematics teaching does not necessarily imply a certain science-philosophical position (for instance of a materialistic shape), although it

must be admitted that some positions are closer to this view than others. In principle, however, there is nothing to prevent an insistence on a formalistic-deductive conception of mathematics, co-existent with endeavours to make mathematics relevant to reality and society.

It is quite as important to stress that with these reflections it is not claimed that teaching can neglect the subject-internal demands for consistency and precision. What is claimed, however, is that these demands alone cannot be sufficient objectives. In this connection, therefore, the crucial point is, how to establish consistency and precision.

VI. The existing role and education of the mathematics teacher

Let us, in order to understand the role that the mathematics teacher plays and therefore must be prepared for by virtue of his own education, have a look at the existing studies in mathematics. Once again the discussion is concerned with Danish material but it should also contain more general elements.

The grammar school teacher in Denmark has taken a university education - at master's level - of 5-7 years duration. The mathematics education at the "old" universities (in Copenhagen and Århus) has not been specifically directed towards a grammar school teacher's function in its building and content but rather towards an education of mathematicians as such, who might alternatively go into research careers, positions in quasi-scientific institutions, libraries etc. In practise the vast majority of graduates have become grammar school teachers (or teachers at teacher training colleges).

The education was earlier an education with mathematics as the major subject plus one or two minor subjects, most often physics and astronomy. In the sixties it became possible to study mathematics full time, while it also became possible and usual to have freer combinations of mathematics with another subject.

To obtain authorization for grammar school teaching today, one's

education is required to consist of two of the subjects included in the grammar school curriculum. Since one of the subjects is a minor subject, authorization is in principle obtained on the basis of a minor's level. Besides the two "proper" subjects - and outside the degree as such - the future grammar school teacher must take a half year course, the so-called "pedagogicum". In this course training in teaching-practise is the most important part, while a smaller subcourse in theoretical pedagogy and psychology fills out the rest. During the university study proper students have in no form met with topics outside their one or two subjects. The philosophy has been (and is) that that part of the teacher's education which takes place at university is an education in one or two scientific subjects, after which the vocational training takes place in another way.

This implies that pedagogic or didactic - not to speak of social - problems have been absent during the mathematics study. Furthermore however, the topics have almost all been purely mathe matical in character. Applied mathematics plays a very humble part and is not required for teaching authorization. Students, however, purely studying mathematics have numerical analysis and probability theory with statistics as compulsory topics, whereas students having a subject besides mathematics do not need to have worked with such topics.)

It may be said that an education of this type corresponds to a teacher function, for which "the good mathematics teacher" (compare the description above, quoted from Glaeser, of the "eminent student") is a person who is capable of inducing insight in the topics of school mathematics under various perspectives and on various abstraction levels, who can make transparent the intuitive as well as the precise content of definitions, theorems and proofs, who can take obscure formulations of the students and give them precise forms, who has a general knowledge of typical fallacies, who can set students' minds at ease in their occupation with the razor blades of mathematics, ans who could never become addicted to turning these blades against any student - in short, a person who is an able representative for

mathematics as a science besides possessing general tractability and consciousness of conceptual hurdles. The particularly competent teacher can furthermore illustrate a number of mathematical points with examples gathered from other subjects or from everyday's practice.

VII. An altered role and education for the mathematics teacher. The described role represents in its own way a reasonable answer to the set subject of teaching students an, on the whole, well-defined and coherent mathematics curriculum-content, by means of which a number of typical physical and technological problems can be given a clear and effective treatment.

As has been discussed earlier in this article, this, however, can no longer be the task. Neither is it any longer obvious that the teacher role must have the described shape.

Firstly it is clear that a mathematics instruction which does not primarily aim at providing a fixed mathematical apparatus for the mental archives of the students, but instead essentially aims at contributing to the students' own building up of mathematical structures for treating given complex problems from reality, demands teachers who are able both to handle such situations themselves and to assist students in doing so. To be able to handle such problem-situations demands more than "the mere" possession of a keyboard of ready mathematical disciplines which can be tested on the situation one at a time. It also demands the capacity to extract from a problem the mathematical possibilities of treating it, possibilities that are not necessarily included in the teacher's reportoire and which therefore must be discovered or even developed. This creation can of course not take place from nothing. The teacher must have had a number of theoretical as well as practical experiences in building mathematical models for non-mathematical phenomenae, in order to have a foundation for creating. From this, however, it does not follow that it is canonically determined which specific mathematical topics the teacher must master.

Secondly, the function of the teacher must include the ability

to cooperate with other teachers in other subjects to arrange teaching situations that can make clear which properties of a mathematical model are mathematical consequences of assumptions about reality, and how changes of these assumptions imply changes in the mode of operation of the model. In addition to this he must give critical assistance with model constructions done by students.

These qualities should serve to demystify the role of mathematics in society, history and in the sciences, in order to contribute towards making it natural for students to use mathematical approaches to complex problems which they meet with in school and later as adults.

The teacher must be able to promote an understanding of the position of mathematics from science-philosophical points of view - not only concretely, with reference to model-building, but also with reference to science philosophy in a broader sense. This demands that the teacher himself possesses comprehension of the fabric and the discipline structure of mathematics - a demand which should not be interpreted as concerning a more or less superficial touch with all the disciplines of the mathematics science.

Among the most important qualities for a teacher in mathematics is that of being able to arrange any teaching situation in such a varied way as to pay attention to the particular social and intellectual background of the individual student. This implies that the teacher must be able to pursue - also from a professional point of view - the same objectives in different ways, on the foundation of pedagogical/psychological and didactic reflections on the actual situation.

In order to be able to carry out such instruction the teacher must be educated according to principles different from the current ones. He must have gone through an education where he himself has acquired experience of problem-oriented work on mathematical models and with problem-directed selection and learning of mathematical theories and methods. (Compare the discussion in section V.) As Wittmann [17] puts it: "But the teacher

is only able to practice freer methods and lead students towards independent mathematical thinking, if his own training has provided him with this kind of attitude - at least to a modest degree."

VIII. The realization of a new mathematics teacher's education.

Usually it will be found that several more or less bright and more or less Utopian ideas about radical changes or just innovations in existing conditions remain in theory without being carried out. For once the situation in this connection is slightly different. At Roskilde University Centre in Denmark there is now established a university education programme directed towards the vocation of a grammar school teacher in mathematics, an education which in several respects follows the principles presented in the preceding sections. The education was started in the autumn of 1974 after having been recommended and planned at Roskilde University Centre and thereafter elaborated by a committee for which I was the chairman. The committee, which was set down by the Ministery af Education, finished its work in the spring of 1974.

The next chapter is devoted to an exposition of the framework and content of this education programme. It seems reasonable, however, to point out in advance that the ideas behind the programme are not new in all respects. For instance similar ideas have been put forward in many articles in the publication series "Materialien zur Analyse der Berufspraxis des Mathematikers" from the University of Bielefeld in Germany.

Apart from this, it is probably necessary to draw attention to the fact that the education programme in question, which has obtained governmental authorization, is directed towards a grammar school that is not (yet?) very much marked by the educational principles discussed above. To this must be added the fact that the strong division of subjects in the Danish grammar school curriculum, both in theory and in practice, makes the individual subjects very autonomous, without much interaction. The mathematics teacher education programme at Roskilde University Centre must, therefore, in its whole be considered as a pragmatic combination

of the more radically experimental ideas, for which I have advocated in the preceding sections, and a number of more traditional ingredients bearing their shape from a regard to the existing grammar school teaching in mathematics and to the tradition of the subject.

B: THE GRAMMAR SCHOOL TEACHER EDUCATION IN MATHEMATICS AT ROSKILDE UNIVERSITY CENTRE (RUC).

I. General remarks on the RUC-studies.

In section VII in chapter A were discussed the tasks which a grammar school teacher education in mathematics should aim at handling.

In order to be able to give an understandable account of the content and structure of the RUC-educations it is necessary to say one or two words about the special educational characteristics of Roskilde University Centre, even if not all the details mentioned deal with the mathematics teacher education in a direct manner.

Roskilde University Centre received its first students in 1972. Its educational structure is very different from what is seen in most other places and consists of three dimensions, none of which must be absent in a description of the studies.

I.1 Study structure.

There exist two main blocks in the educational pattern of RUC, the basic education programmes and the further education programmes.

There are three basic education programmes, one of natural sciences, one of humanities and one of social sciences, all of which are prescribed two years' duration and none of which give any independent professional authorization, or, put in another way: the basic education programmes do not suffice to gain university degrees.

After having satisfactorily taken a basic education programme a student can as his major possibility continue his studies by one of the further education programmes at RUC. Here particular attention should be payed to the longer further education programmes - which are prescribed a total duration of 5-5½ years (including the basic education) - since the grammar school educations are all of that type.x) No formal restrictions are put on the choice of the basic education following which the individual student begins a certain further education programme but in practice most of the students following such a programme have taken the most relevant basic education and students not having done so must foresee extra work or prolongation af study.

Thus the vast majority of grammar school teacher students with subjects from natural sciences have their basic education in natural sciences.

To summarize the formalities: a university degree from RUC comprises a completed basic education together with a completed further education.

All the education programmes have ministerial approval, meaning in effect that graduates with resultant degrees are eligible to apply for a number of posts for which special authorizations are required.

x). It should be pointed out that the committees which completed the elaboration of the grammar school teacher education programmes also completed the plans on teacher education programmes of a shorter duration (3½ years, including the basic education) for primary schools. They were planned in such a way that a primary school teacher educated at RUC could - practically without having "wasted" any time - suppliment his education to that of a grammar school teacher's by 2-2½ years additional work. For political reasons, however, the minister of education in the then government did not wish to allow the establishment of such an education at RUC.

Any further education programme is built up of modules. By a module is understood a certain set of requirements to content which an average student should be able to fulfil by studying over the total extent of one semester. The studies within a module need not take place connectedly , even if they often do so in practice.

The grammar school education programme which is our interest here, consists, after a basic education programme, of two subjects, each containing three modules[x)], and a seventh module, the so called job-module. The concept of a module serves primarily to indicate requirements to pieces of work and a way of measuring its extent. There are no formalities preventing a student from studying within the range of more than one module (or more than one subject) at the same time.

For each module the student must take an examination, at which external examiners participate. When a student has passed examinations for all the modules (and of course of a basic education), the education as such is complete, and the graduate gains the degree "cand. mag." (candidatus magisteri).

I.2. The contents of the studies.

The peculiarity of the education programmes of Roskilde University Centre is not primarily their structural building up described in section I.1, but rather the content (and the organisation) of the studies.

In the basic education as in the further education programmes, the principal study form is problem-oriented and project-organized. This means that students work in groups, each under the guidance of one or more teachers, with a project the object of which is a complex problem, typically of social origin/relevance, and which

x) The two subjects, therefore are of equal importance, none of them being a major or a minor subject.

does not immediately refer to any specific area of science. Only after a sequence of "narrow-downs", simplifications, choices of view-points, and ensuing formulation of the resulting problem are scientific theories, methods and techniques actualized.[x]

This rather radical problem-orientation towards problems of social origin/relevance, where the subjects appear not before, but after the problems, represents the key to the understanding of the special character of RUC: Project-work is not "only" a technical-teaching device, which allows students to learn what is usually learnt but under more inspiring and motivating forms. The problems dealt with are often also interesting from a research point of view and have often not been treated previously in their actual form by other people although they may, of course, reproduce or explicitly apply results already gained by others.

The other important feature of project-work is that students themselves having consulted their teachers formulate and narrow down the complex problems to be treated, take care of the work planning needed, search for literature and work up the problem. Altogether the work goes on quite like a research project proper, which often actually is the case.

Here an urgent question poses itself: How can it be meaningful to operate with education programmes on traditionally labelled subjects, when the material actually studied is not constituted by subject categories?

The answer consists first of all in noting that education at grammar school <u>is</u> divided into subjects, whether one likes it or not. Secondly, the role of a subject might be described as performing a selection within the set of problems in such a way that students with a given subject only deal with problems that give occasion for a substantial involvement of views, theories and methods from the subject (as understood in its traditional sense) in question. This puts forward a subject concept which is different from the one which lays the main emphasis on the subject as a total edifice of knowledge which in whole or in part should be the object for study.

x) As to the actually dialectic character of this process, cfr. chapter A, section V.

Normally a project runs within one semester, but it would occupy all the time of a semester only rather seldom. To support or to supply the project-work students take courses, or participate in seminars and colloquiums.

The completion of a project is followed by the production of a project-report, which is manifolded and submitted to critical discussion among teachers and students, and if possible external opponents, at a so-called presentation-seminar.

From an organizational point of view the principal study-unit is the house. A house consists of 40-100 students and 4-10 teachers, who are all kept together during a longer period (a couple of years or more). Project groups are normally formed inside the house just as the project teacher(s) normally come(s) from the same house too. In the course of the year 1976 institutes will be established, a fact that will alter the organizational structure quite a lot.

Since it is the natural science basic education programme that typically makes the entrance to the mathematics teacher education, I shall make a few remarks about it here.

I.3 The natural science basic education programme.

The one half of the time in each semester of the natural science basic education programme is devoted to project-work. The project-work must in the four semesters at disposal deal with problems concerning the following problem-areas:

1. semester: working conditions, energy, resources, pollution.

2. semester: internal natural science matters.

3. semester: communication of natural sciences to various receiver groups, especially in the school-system.

4. semester: no restrictions on content.

The second half of the time is occupied by courses amongst which are general courses in mathematics, statistics, physics, chemistry, bio-subjects, geo-subjects etc. as well as more special courses established ad hoc to support the project-work. Each student must in the course of his basic education take eight courses, almost of free option except for some combination requirements.

At the end of his basic education the student must choose one of his projects to present at an examination, where he is examined on the project-work, with participation of external examiners.

II. The grammar school teacher education in mathematics.

II.1 General remarks on the grammar school teacher education programmes.

It is an important property of the grammar shool teacher education programmes at RUC - in contra-distinction to other such programmes in Denmark, which, as earlier mentioned, are directed also towards other professions than that of teaching - that subject matters proper are studied integrated with pedagogical/didactic matters. The question is not, therefore, about two different, isolated types of activities, because problems concerning subject matters are given a school-oriented treatment too. As a background for doing this, not only pedagogical-psychological, and in a narrower sense subject-didactic, reflections are supposed to take place but also more general considerations on the function of one's subject in school and society. The modules of the grammar school teacher education programmes are, therefore, called subject-pedagogical modules. In the seventh module, the job-module, general, not subject-specific school-matters are studied but the module consists firstly of practice. Each student must go through 150 hours of practice, of which 120 hours must be teaching practice in school. In contrast with the other grammar school teacher education programmes in this country this practice constitutes a part of the study proper and takes place in close

connection with the theoretical aspects of it.

II.2 The mathematics teacher education programme.

The grammar school teacher education programme in mathematics displays the same general features as do the other teacher education programmes at RUC. Thus the education - besides mathematics - includes another freely chosen subject.

It is important to stress that the problem-orientation as to the mathematics programme is to be understood in the previously mentioned thorough sense, where the concrete mathematical disciplines, theories and methods dealt with are actualized by the complex problem at issue, under consideration of some requirements listed later in this section. So the problem-orientation is neither to be understood in the well-known meaning according to which it is equivalent to puzzle-orientation, or in the meaning which has become quite widespread in the last few years, in which emphasis is laid on problems which can contribute to elucidating a given mathematical building of theory under consideration (be the problems ever so realistic).

While in the natural science basic education programme a certain ratio between project-work and course work is fixed, no similar provision holds for the grammar school teacher programme. Instead the rule applies that to the extent that requirements to an individual module can not be fulfilled by project-work alone, these must be supplied by course or seminar work.

In the beginning of his study and under guidance of his fellow students and teachers, the individual student makes a working-plan, outlining the way in which he intends to fulfil the requirements of the education. This plan is subject to adjustment each semester.

In the following the education programme in mathematics is described with reference to "Government Notice of October the 7th 1974 on the grammar school teacher education programmes at Roskilde University Centre" [18], which constitutes the formal, ju-

ridically binding provisions, recommended by the ministerial committee earlier mentioned and later issued by the Ministery of Education.

The grammar school teacher education programme in mathematics has the following objectives:

The student should acquire a background as a future teacher, for

a) assisting grammar school pupils in building, analysing, evaluating and applying mathematical models. This should include the ability to impart a sufficient basis to his pupils to enable these to work with mathematical models as well as the ability to impart a sufficient knowledge to demystify the scientific and social positions of mathematics;

b) assisting grammar school pupils in inventing and carrying through inference in deductive sequences;

c) bringing about the science-philosophical state (in a broad sense) of mathematics, inside the subject as well as in connection with other areas of science and society, in a way suitable to the grammar school level;

d) mastering a series of different teaching forms and evaluating their applicability to actual teaching situations;

e) regarding cognitive problems at various stages of mental development and in various social groups aiding the acquisition of insight into mathematics;

f) bringing about and enriching a current debate with pupils about their mathematical reasoning;

g) acting as a mathematical guide for pupils and teachers of other subjects;

h) working scientifically with methods and theories of mathematics, in order to be able to follow the development of and debate on the subject.

These objectives are intended to be fulfilled by the following two sets of requirements to the content of studies:

In the course of the 1½ years at disposal for specifically mathematical studies (which may very well be spread all over the total range of his further education) the student must have worked with the following five nodal points:

1. Mathematical model building, including stoctastic as well as deterministic models, and independent construction of "new" models as well as evaluation of existing ones.
2. The position of mathematics from science philosophical points of view, including
   - the sense in and extent to which mathematical arguments within a model can be interpreted as meaningful and true statements about that reality which the model aims at describing,
   - the relationship between mathematical ways of thinking and deductive processes in other connections,
   - the way in which mathematical knowledge is generated by, among other things, an interaction between the internal dynamics of mathematics and its modelling functions, and furthermore the points and ways in which mutual influence of the development of mathematics and processes in society actually has taken place.
3. Axiomatic treatment of areas of mathematics.
4. Cognitive problems concerning acquisition of mathematics at various stages of mental development and in various social groups.
5. Various working forms for mathematics teaching at school, considered as a means of providing pupils with a fund of comprehended accomplishments.

The work with the nodal points must be done in such a way that the individual student in the course of his study has been occupied with the following eight topic areas:

1. Logic and set theory, studied in such a way and to such an extent that the student can clarify the significance and function of these general concepts in the building up of mathematics teaching and in the treatment of mathematical contexts.
2. The structure of the number systems, studied in such a way and to such an extent that the student has acquired familiarity with an axiomatic treatment of the natural numbers and

with a constructive extension of them, via the rational numbers and real numbers to the complex numbers, and furthermore largely can relate these constructions to such aspects of a non-mathematical character that play a role in the cognitive formation of the concept of number.

3. Linear algebra, studied in such a way and to such an extent that the student obtains familiarity with the fundamental concepts of linear algebra, so that he can apply this knowledge in mathematical contexts where linear structures appear.

4. Geometry. One or more general geometries studied in such a way and to such an extent that they in a more general framework can elucidate and perspectivate those aspects of euclidean geometry which are relevant to teaching purposes.

5. Theory of probability and statistics, studied in such a way and to such an extent that the student is able to realize and to treat these concepts, when they appear in stochastic models for phenomenae from reality.

6. Real functions of one or several variables, studied in such a way and to such an extent that the consequences of continuity -, differentiability- and integrability-properties of such functions in model contexts can be realized and treated by the student.

7. Differential equations, studied in such a way and to such an extent that their role in deterministic models can be realized and treated by the student.

8. Topological structures, studied in such a way and to such an extent that the student can realize and treat this aspect of compound problems dealt with in the topic areas 1.-7.

Obviously the nodal points rather than the topic areas represent the real innovation by the RUC-education, since the nodal points frame the universe of project possibilities. The topic areas - which in their titles but perhaps not in their characterization of content are quite traditional - constitute a structure "orthogonal" to the nodal points. This structure is supposed to take into consideration the fact that mathematics in Danish grammar schools still have mostly traditional outlines. The topic areas might be considered as representing a safety net, in the sense that topic areas acquired by an individual student by means of project-work need not be subject to further treatment. Such requirements only appear if a given topic area is not acquired by an individual student by means of project-work. In the working-plan of the individual student is indicated which topic

areas are intended to be treated by project-work and which independently.

Let it be said that the untraditional demarcation of the topic areas should not be interpreted as equivalent to the neglecting of their structure, consistency and stringency, for instance in overriding proofs or in being content with plausibility arguments, for the power of mathematics as an instrument for treating the outside world relies primarily on its structural properties.

The formal coupling between the requirements to the nodal points and the requirements to the topic areas is expressed in their combination in the modules. The three mathematics modules are fixed in this way:

Module 1: The nodal points 1,2,3,4,5. This module is best conceived of as a "module of breadth", where the totality of nodal points should promote a broad idea of the range of the education[x].

Module 2: The nodal points 1,4,5. Shortly speaking this module is concentrated on mathematical models, studied, among other things, in the light of cognitive problems concerning pupils' work with models and of the specific working forms applicable in school to such work.

Module 3: The nodal points 2,3,4,5. This module is concerned with the position of mathematics from science philosophical points of view in the light of the aspects represented by the nodal points 4 and 5.

The modules need not necessarily be studied in the order indicated above, cfr. the remarks in section I.1 of this chapter.

x) If the education programme for primary school teachers earlier mentioned had been carried out, this module would have made up its (only) mathematics module.

The formal relationship between nodal points and topic areas is established in that the individual student associates to each of the three modules those topic areas with which he in connection with his project-work or as a supplement to it (by means of courses etc.) has worked. Altogether each of the topic areas must be associated to a module but it is a matter of the student's own planning (under teacher guidance) to fix the actual mapping. Each module examination consists of a written paper on one topic area and of an oral part on the nodal points relevant to the module in question. The oral part, having its basis in a description (written by the student) explaining by what activities the student claims to have fulfilled the requirements to the module, will most often consist of a discussion of a main project, while the written examination paper will draw its material from work which has been a part of some project-work or from course-work without relation to any project-work. Those topic areas which are not included in the module examination are submitted to internal examination, i.e. the teacher, without participation of external examiners, decides whether or not the given student fulfils the requirements set to those topics.

Possibly it is a little difficult to see, on the basis of the rather formal presentation above, how studies are carried out in practice. The next section will therefore contain a description of how a group among the first mathematics students (having finished their basic educations in summer 1974 and having started their mathematics studies autumn 1974) have worked in the first part of their further education.

II.3. An authentic example.

The students in question work (with students from other subjects) in a house, where among other things the application in society, primarily in Danmark, of natural sciences and mathematics is studied. So it seemed natural to these students to find out what applications of mathematical theories and methods are actually done in the Danish society. This was primarily due to an explicit

interest of the students in investigating the application role of mathemathics, in order to prepare themselves to emphasize this role in their future teaching. They therefore started their project work by detecting where in Danish public administration and industry mathematical models are actually used and not only constructed.

On the background of the results from this the students chose together with their teacher (the author of this article) to make an evaluation of a macro-economic model of Denmark, generated by a quasi-public research and advisory agency which is used by the government and parliament in connection with the estimation of the effect of fiscal policies. The model is a non-linear but still rational equation model, consisting af 56 equations, six of which contain coefficients estimated "outside" the model, while the remaining equations virtually only present definitions and identities.

After having studied the official publications on the model the students decided, as a main task, to investigate how to evaluate mathematical models on economic, and hence social, processes and forecasting models in particular. In connection with an extensive, summarizing evaluation of the model, a subproblem was formulated the purpose of which was to serve to elucidate the mathematical properties of the model as a part of an assessment of its quality. This subproblem consisted in investigating the sensitivity of the model under small changes in its specifications, i.e. the "known" terms of the equation system of which some represent fiscal policy control variables, others represent variables beyond political control, while others again are variables determined as solutions to the equation system for earlier periods. Or mathematically formulated: can the solution vectors (to the extent that they at all exist) to the equation system be considered as values of a continuous function of the in-put-specifications?

To answer this question - and in general to understand the whole character of the model - the students had to read text books on linear algebra and real functions of several variables.

The main tool was the implicit-function-theorem, and possible points of singularity for a truncated Jacobi-matrix whose elements were functions were to be found.

So it is seen that the increasingly sharpened narrowing down of the problem, characteristic of project-work, led the students - in order to qualify themselves to tackling the question they had posed - to learn usual text book material on well-known topics. Since this learning took place in a context meaningful to the students it - over and above being better motivated - became more effective than it would have been if the students, for instance following directions from me, had prepared themselves from the very beginning for the project-work by reading text-books. And I would guess that not many students in the world will have such a direct, "physical" familiarity with calculating the determinant of a 56x56 matrix with functions as its elements, a problem where computers do not offer very much help.

This project-work played, together with the topic area "linear algebra", the main role in the fulfilment of module 2 (while the topic area "real functions" is to be associated with one of the other modules). In addition to this, the students have in a course, separate from the project-work, studied logic, set theory and a constructive building up of the number systems, corresponding to the topic areas 1 and 2 from standard text books and from lecture notes written by me. Therefore module 2, for <u>these</u> students was fulfilled by the activities mentioned, while perhaps quite other substances and quite other associated topic areas will do for other groups of students and teachers at another time.

II.4 Concluding remarks.

To summarize: In establishing a <u>fully</u> problem-oriented, project-organized mathematics teacher education one must regard three conditions:

<u>Condition 1</u>: If problem-oriented project-studies in mathematics

are not degenerate to a mere technical-teaching device which in an effective way motivates students to learn a fixed curriculum content, it is necessary to accept the projects to be selected according to their potential for being exemplifying and not according to their potential for teaching students a fixed curriculum-content.

Now it is fairly evident that any piece of mathematics can in principle be treated by metatheoretical projects of a science-philosophical, science-historical or didactic character, where internal mathematical matters form the object of the project-work. In project-work that is not of this sort but turned outwards from mathematics, there will be disciplines occupying a large amount of room inside the universe of mathematics, which vill be actualized only very seldom. This cost is necessary if projects of the latter sort should be weighty in a project-organized mathematics education.

Another considerable - and for many people not very acceptable - cost is that the content common to all students in the education - the canonical content - will be very limited.

Condition 2: It is vital, if the experiences induced by a project-organized study of mathematics are to be productive (have a transfer-value) in other problem-situations, that the mathematical concepts and results applied are acquired by students under consideration of the internal building principles of the subject. One cannot just take the cream off the milk. The power of mathematics relies particularly upon its structural and generalistic properties which therefore must not be neglected. The mathematical horizon of the individual student must be connected and fairly consistent.

Condition 3: What requirements to the type and treatment of project-problems follow from the condition that in working with them, mathematics should be learned in such a way that the gained knowledge is productive in other situations? Our experiences do not indicate any particular requirements to the type of problems but rather to the treatment of them. For if a problem is of a certain breadth and complexity, one must

face up to being able only to deal with a subproblem, resulting from a - typically quite strong - narrowing-down of the original problem.

From what has been described in the previous sections it is seen that the teacher education programme in mathematics at RUC does not quite fulfil the first part of condition 3, since the education programme actually does contain requirements to curriculum-content. However, most of the fundamental characteristics of a problem-oriented, project-organized study are intact.

It is too early to put forward more detailed comments on the advantages and shortcomings of the education programme described, since we, for obvious reasons, have not sc far acquired sufficiently extensive experience on its concrete effects, neither the internal (regarding the students as students) nor the external (regarding the capacity of the future graduates for mastering their teaching-tasks according to the intentions upon which the education programme is built) ones.

Some points may, however, be brought up here.

1.
Compared to the education programmes at other Danish universities having mathematics as the major subject, there is no doubt that the RUC-programme typically implies a smaller curriculum-content, in a narrow mathematical sense, than do the others. (If , on the other hand, a comparison were made between programmes with mathematics as the minor subject, which suffices for gaining teaching authorization in grammar school, the picture is very different.) It is possible, or even probable, therefore, that students will not meet with the more advanced buildings of theory, resting on a hierarchy of concepts and results, very often. This weakens students' possibilities of being confronted with the internal research process of mathematics because they will seldom reach the front of a discipline. This is not a fundamental consequence of the study-form, which on the contrary sets students' absorption in the problems dealt with, but rather a consequence of the variety and breadth of the requirements

to the education in combination with its relatively short duration (fixed by the Ministery of Education).

In this connection, however, it should be said that it is actually possible for students to grasp large quantities of mathematics in shorter time than is normally seen, if the actual piece of mathematics is involved in project-work. On the other hand it seems that proficiency in handling details requires supplementary excercise of a more traditional character. The crasping of material which has no direct relationship to project-work takes significantly longer time than is necessary for project-generated mathematics material. In return, however, students insist on understanding not only the details but also the outlines and constituting structures even of material which is not project-generated.

Finally one must keep in mind that the education programmes with mathematics as a major at other universities should be considered as mathematician-educations rather than grammar school teacher educations proper.

2.

It is obvious that the RUC-education in order to gain success on its own premisses requires more from the students as to working morale, capacity for independent reading of scientific literature, critical potential, intellectual maturity etc.., than is required at universities where the material is to a larger extent well-prepared. These larger requirements partly follow from the general character of the RUC-way of studying, and partly from the fact that the mathematics teacher education contains a large variety of extensive demands to a connected comprehension of mathematics and its roles, without too much time for answering these demands.

With a majority of the first students having started with mathematics at RUC, a far-reaching capacity for answering the demands at issue can be found. From this, however, one cannot infer too bold conclusions, since these students are pioneers in being members of the first intake of a new-born university

as well as being its first mathematics students. These students can hardly be considered as typical, and perhaps future students who to a decreasing degree can influence the establishment of traditions of the education, will do less good without necessarily being of poorer "initial capacities".

It is still too early to say definitely whether or not the demands can in practice be answered by an average student.

3.
Among the difficulties which might show a certain persistence are the difficulties in carrying through a really integrated work with, on the one hand, the application role of mathematics and, on the other hand, the position of mathematics in grammar school, at least when the question is about applications requiring more advanced mathematics than can be dealt with at grammar school level. This might imply that the didactic treatment in such application-oriented projects would possibly seem general or superimposed. The requirement that all matters must be regarded in a cognitive and working-form-oriented light is perhaps too rigorous but at this point also it is too early to put forward any definite evaluation of possibilities and limitations.

4.
Before one tries to make an assessment of the educational experiment here presented, one should keep in mind that such an assessment should not consist in a mere comparison of the formal organization and regulation of other grammar school teacher education programmes in mathematics but in an evaluation of the resulting capacity of the graduates to fill the role demanded by a changed mathematics teaching in which is comprised the capacity for not being a sacrifice for future changes but a participant in them.

Mogens Niss, June 1st 1976

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[18] Government Notice of October the 7th 1974 on the grammar school teacher education programmes at Roskilde University Centre, Danish Ministery of Education.